Floating Orbits and Ergoregion Stability of an Exotic star-Black hole system.

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Introduction: A floating orbit is a metastable orbit around dense rotating astrophysical bodies wherein the orbiting body avoids orbital decay by stealing rotational energy from the central body. Normally, the orbiting body would emit gravitational waves as it orbits the central body, with the waves being directed toward the central body and outer space. The emission of these waves decreases the energy of the system and lessens the orbital radius. In the case of a floating orbit, the waves that are directed toward the central body interact with the central body's ergoregion, which is a corotating region of spacetime around a dense, rotating central body.

The waves take energy from this ergoregion, travel back toward the orbiting body, and impart their energy to the body. This process is called superradiant scattering (SRS), for the returning gravitational waves possess more energy they had when they left the orbiting body. These impinging waves balance the total energy loss by the orbiting body, thereby keeping the orbiting body's radius stable while decreasing the revolution of the central body.

These orbits are important for gravitational wave research and the further confirmation of Einstein's theory of General Relativity. Einstein's equations show that the gravitational interaction of two bodies should emit gravitational waves. Unfortunately, these waves are of low amplitude and couple to matter weakly, therefore they have been extremely difficult to detect. However, if a floating orbit can be shown to exist theoretically and if one is discovered in nature, then the system would emit gravitational waves at a regular frequency. With a constant source of waves interacting with the detectors, random noise and other anomalies can be eliminated and the general trend of the radiation would be easier to detect and study.

The focus of my project was to investigate whether by altering the previous solution employed by Kennefick and Glampedakis[1], which they applied to the case of a central black hole, to represent a central exotic star and employ a test developed Richartz et.al. [2] to see if our star would permit SRS. Furthermore, by taking the Hartle process into consideration, we intend to show that a floating orbit around an exotic star is theoretically possible.

Superradiant Scattering and the Penrose Process: Einstein showed in his general relativistic field equations that a non-spherically symmetric dynamical system emits gravitational radiation. This radiation travels at the speed of light, possesses both translational and angular momentum, and is quadrupole in nature. The emission of this radiation decreases the energy of the system and the orbital radius, and eventually the orbiting body inspirals.

In the case of floating orbits, the inspiral due to gravitational wave emissions can be prevented by SRS. In SRS the waves lost to space are balanced by the waves that are emitted from the exotic star toward the orbiting body, in a process is called the Penrose process.

In the Penrose process, an object enters the ergosphere of a dense astrophysical body moving in the prograde direction, and the object breaks into two pieces, with one piece being directed toward the central body and the other piece directed outside of the ergosphere. Within this ergosphere, the spacetime and all matter is being dragged around at a velocity greater than light. Consequently, the object while in the ergosphere possesses a velocity greater than the local spacetime due to the addition of its previous velocity to that of the ergosphere.

The inward going piece possesses a velocity less than the local spacetime and causes the central body to decrease its rotational energy in order to accelerate it to the velocity of the spacetime. While the outward going piece has a velocity that is greater than the initial object and escapes the ergosphere with more energy than the initial body had. In our case the initial object would be a gravitational wave and after this process it would impart its kinetic energy on the orbiting body thereby balancing the energy the body lost to space.

Wronskian Calculation: Richartz et. al. argued that the Wronskian of the solutions of the Klein Gordon equation can determine whether a system possesses SRS. For a system to have SRS, |R|, the reflection coefficient of the wave solution must be greater than one. From their equation the following conditions of SRS can be found: $iW(f,f^*)|_{\xi_0} - 2\int_{\xi_0}^{\infty} \Gamma(\xi)|f|^2 d\xi > 0$. Therefore, either $iW \geqslant 0$ and $\Gamma \leqslant 0$ over the interval, or $iW \leqslant 0$ and $\Gamma \geqslant 0$ over the interval. Moreover, if $iW \leqslant 0$ and $\Gamma \geqslant 0$ then SRS is impossible and here is where we inserted our general solution for f.

The form of f that we used was a modification of the wave solutions employed by Kennefick and Glampedakis. Their solutions were came out of the radial part of the Teukolsky equation, which is a equation derived from Einstein's equation describing a Kerr black hole. The altered equations are as follows:

$$R^{in}_{lm\omega} \to \Delta^2 e^{-ikr^*} + A R(r) e^{-i\omega r^*} \quad for\ r \to r_+$$
$$r^3 B^{out} e^{i\omega r^*} + r^{-1} B^{in} e^{-i\omega r^*} + A R(r) e^{-i\omega r^*} \quad for\ r \to +\infty$$
$$R^{up}_{lm\omega} \to C^{out} e^{ikr^*} + \Delta^2 C^{in} e^{-ikr^*} \quad for\ r \to r_+$$
$$r^3 e^{i\omega r^*} \quad for\ r \to +\infty$$

. The Wronskian of these two solutions, for $R(r) = r^{-n}$, for any $n \in \mathbb{R}^+$, are:

$$W(R^{in}_{lm\omega}, R^{up}_{lm\omega})|_{r \to r_+} = 2 i\omega C^{out}(a^4 - M^4) \quad for\ any\ n$$
$$W(R^{in}_{lm\omega}, R^{up}_{lm\omega})|_{r \to +\infty} = 4 i\omega (B^{in} + A) r^2 \quad for\ n = 1$$
$$2 i\omega B^{in} r^2 \quad for\ n > 1$$

. If we go by Richartz et. al. then we will only experience SRS if the Wronskian contains a negative imaginary part. Therefore we would expect to see SRS when evaluating at r_+ , if $M > a$, which is the case in general, and if when evaluating at infinity, A is negative and greater in magnitude than B^{in} for the $n = 1$ case and we will never see any SRS if $n > 1$.

It should be noted that the sign of the Wronskian changes depending on the ordering of our independent solutions, which is an ambiguity in Richartz et. al.'s condition for SRS. Fortunately, Kennefick and Glampedakis provide solutions for which they have proven the existence of SRS utilizing their code that calculates the full Teukolsky solution for a particle orbiting a massive black hole. Since we know that SRS exits for their functions R^{in} and R^{up} , we conclude that correct ordering for the application of Richartz et. al.'s test is one in which we have the order $W(R^{up}_{lm\omega}, R^{in}_{lm\omega})$. We find that, even with our more general solution, designed to include the situation where the central black hole becomes a material body without an event horizon, the sign of the Wronskian remains unchanged, thereby showing the possibility for SRS, unless $A<0$ and $|A|>|B^{in}|$ in the $n=1$ case and SRS is possible, in general, for any $n>1$. However, SRS is not possible for the solution evaluated at r_+ .

By placing our solutions into the equation for wave emission, we do find that SRS is possible for the solution evaluated at infinity so long as $A<B^{in}$ in the $n=1$ case and is possible for any $n>1$:

$$|R|^2 = 1 + 2(B^{in} + A) r^2 \quad for\ n=1$$
$$1 + B^{in} r^2 \quad for\ n>1, \quad r \to +\infty$$

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Ergoregion Instability: The wave emission from an exotic star is potentially more potent due to ergoregion instability. In this process, an exotic star undergoes the Penrose process and the inward wave propagates back toward the center of the star. When this wave hits the center it continues in its trajectory since there is no horizon to absorb it. As this wave makes its way back toward the ergosphere it undergoes the Penrose process. This self-perpetuating Penrose process can radiate energy away from the exotic star far faster than in the black hole case. The faster the rotation the greater the ergoregion instability and consequently, the longer a floating orbit can be sustained[3].

As outlined lined by Cardoso et. al., the timescale of ergoregion instability can be from a few seconds to weeks depending on the mass and rotation of the central star. By equating their equation of the time scale ergoregion instability, which details how long the Penrose process will last, and another equation of inspiral, boundary conditions on the masses of the bodies can be established:

$$m > 7.581\times10^{22}\,kg \quad for\ M = M_\odot$$
$$3.783\,kg \quad for\ M = 10^6 M_\odot$$

which illustrates that a massive central star is needed for a floating orbit. However our exotic star is rotating nearly 300 times that of the star in the Cardoso et. al. paper, which suggests that it would last long enough to sustain a floating orbit.

Lastly, the Hartle approach could also permit a floating orbit. In this process, the orbiting body raises a tide on the ergosphere, but due to the rotation of the central body superseding that of the orbiting body the tide is displaced an angle in the prograde direction. The tide pulls on the orbiting body and accelerates it, while decreasing the rotation of the central body. If this increase in orbital energy balances the energy lost to gravitational waves then a floating orbit is possible.

Conclusion: In summation, we have argued that our solutions to the Wronskian, despite being positive, should display SRS for exotic stars due to the results from Kennefick and Glampedakis paper, and the a floating orbit should be theoretically possible for our system. And showed that our exotic star should radiate more gravitational energy than an similarly massed black hole do to ergoregion instability. Future research will involve actually running the code from the Kennefick and Glampedakis paper to see if SRS can be shown to exist for this system. If the calculations prove fruitful then the energy reflected back at the orbiting body will be checked against the energy lost to gravitational waves. So long as the gained energy supersedes the lost energy, a floating orbit should exist. Lastly, the physical nature of the exotic star would be checked against known star structures and if shown to be in accord with known stars, then a floating orbit could be shown to be possible in nature.

References: [1] K.G. and D. K. (2002) *Physical Review D 66*, 1-33. [2] Richartz et. al. (2009) *arXiv:2317v2 [gr-qc]*, 1-4. [3] Cardoso et al.,(2008) *arXiv:0709.05322v2 [gr-qc]*, 1-14.